



**LON-CAPA**

**Mathematik in LON-CAPA**

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SourceTalk Tage Göttingen

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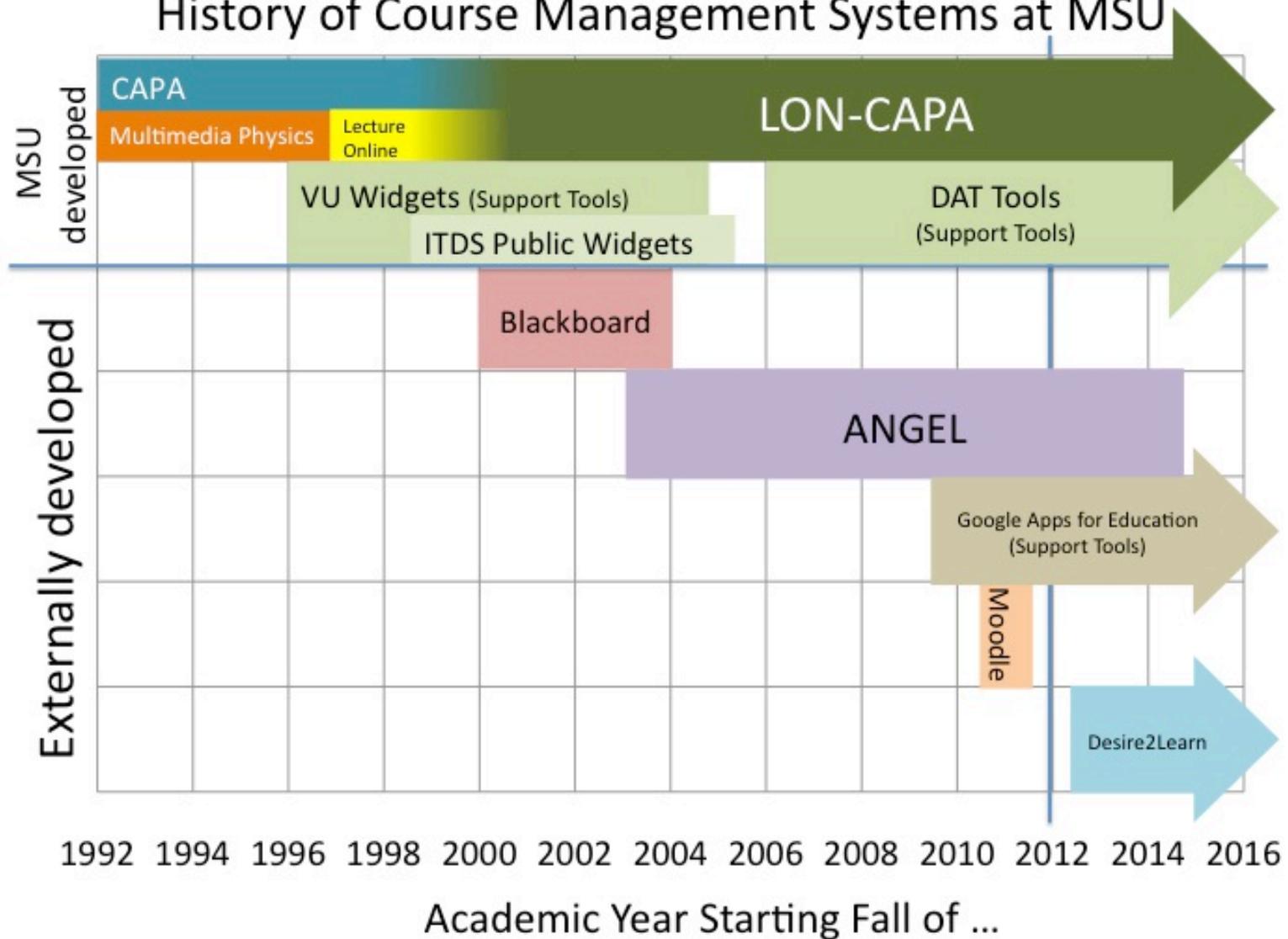
# LON-CAPA



- Seit 1992
- 160 Partnerinstitutionen
  - 94% im Hochschulbereich
- 150,000 Einschreibungen pro Semester
- 440,000 Lehr/Lernobjekte
  - darunter 200,000 Aufgaben
- 73,000,000 gelöste Aufgaben seit 1999
- Open-source

# LON-CAPA bei MSU

## History of Course Management Systems at MSU





# Grundprinzip für Aufgaben

- Verschiedene Studierende bekommen verschiedene Version der gleichen Aufgaben
  - Verschiedene Zahlen, Bilder, Optionen, Formeln
- Aufgaben-Grundstruktur XML
  - Eingebettet: Perl, LaTeX, etc.

# Mathematik in LON-CAPA

- Mathematische Ausgabe:
  - Typensatz
  - Graphen
- Erzeugung mathematischer Probleme:
  - Algebra/Analysis
  - Statistik
- Mathematische Eingabe:
  - Numerisch
    - Numerische Stützstellen
    - Symbolische Äquivalenz
    - Prüfung auf Eigenschaften
  - Grafische Eingabe
  - “Bubble sheets”
  - Clicker



# Mathematische Ausgabe

- Typensatz: LaTeX kann in HTML-Material eingebettet werden

Text Block Delete?  Edit Math  [Greek Symbols !\[\]\(f8e7be3c2bd30232a05cdc54a8b2d22a\_img.jpg\)](#)

Rich formatting »

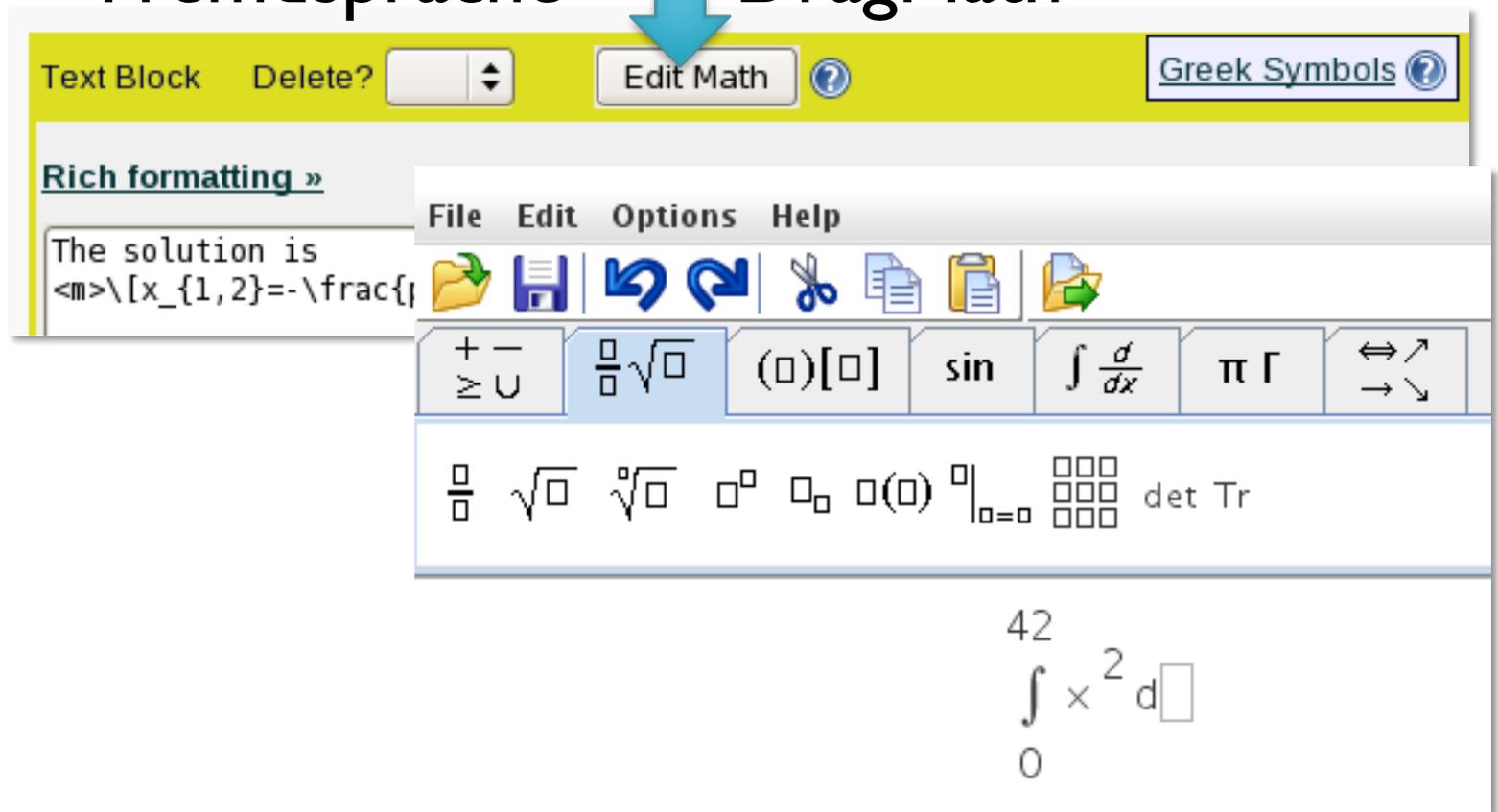
The solution is  
`<m>\[x_{1,2}=-\frac{p}{2}\pm\sqrt{\left(\frac{p}{2}\right)^2-q}\]`</m>

The solution is

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

# Mathematische Ausgabe

- Editor für Autoren mit “LaTeX als Fremdsprache”  DragMath



The screenshot displays the DragMath editor interface. At the top, there is a yellow toolbar with buttons for "Text Block", "Delete?", "Edit Math", and "Greek Symbols". Below the toolbar, a text input field contains the text "The solution is" followed by the LaTeX code `<m>\[x_{1,2}=-\frac{1}{2}\]`. A menu bar with "File", "Edit", "Options", and "Help" is visible. The main toolbar contains various mathematical symbols and operators, including  $\frac{\square}{\square}$ ,  $\sqrt{\square}$ ,  $(\square)[\square]$ ,  $\sin$ ,  $\int \frac{d}{dx}$ ,  $\pi$ ,  $\Gamma$ , and arrows. Below the toolbar, a preview window shows the rendered LaTeX expression  $\int_0^{42} x^2 dx$ .

# Mathematische Ausgabe

- Ausgabe kann “on-the-fly” randomisiert werden

Script

```
$k=&random(2,5,1)
```

Insert:

Text Block   

[Rich formatting »](#)

```
What is the derivative of  
<m eval="on">\[\frac{1}{k}x^k\]</m>  
with respect to <m>$x$</m>?
```

What is the derivative of

$$\frac{1}{3}x^3$$

with respect to  $x$ ?

What is the derivative of

$$\frac{1}{5}x^5$$

with respect to  $x$ ?

# Mathematische Ausgabe

- `<algebra>`-tag zur Formatierung von Ausgabe aus Computer Algebra Systemen
- Beispiel: `$formula="a*x^5"`

Text Block    Delete?     Edit Math 

[Rich formatting »](#)

What is the derivative of `<algebra>$formula</algebra>` with respect to x?

What is the derivative of  $a \cdot x^5$  with respect to x?

Tries 0

# Mathematische Ausgabe

- “One-source, multiple target”
  - Online:

The solution is

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

- Druck (dynamisch erzeugtes PDF):

The solution is

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

# Mathematische Ausgabe

Kein Online-Ausgabesystem für LaTeX gefunden, das für alle Browser und alle Ausdrücke gut aussieht:

- Symbol-Font: tth – funktioniert gut, aber hässlich
- Inline-Images: MimeTeX – Skalierung
- JSMath: JavaScript – eingeschränkte Funktionalität
- Neu: MathJax: JavaScript – vielversprechend, braucht noch mehr Tests
- Innerhalb LON-CAPA auf mehreren Ebenen konfigurierbar, welches System verwendet wird

# Mathematische Ausgabe

- MathML?
- Ja, warum eigentlich nicht?
- Weil's einfach in der Praxis nicht noch funktioniert, deshalb nicht
- Viele Sachen sind so wunderschön und elegant, bis sie für 800 Autoren und 150,000 Studierende pro Semester funktionieren müssen

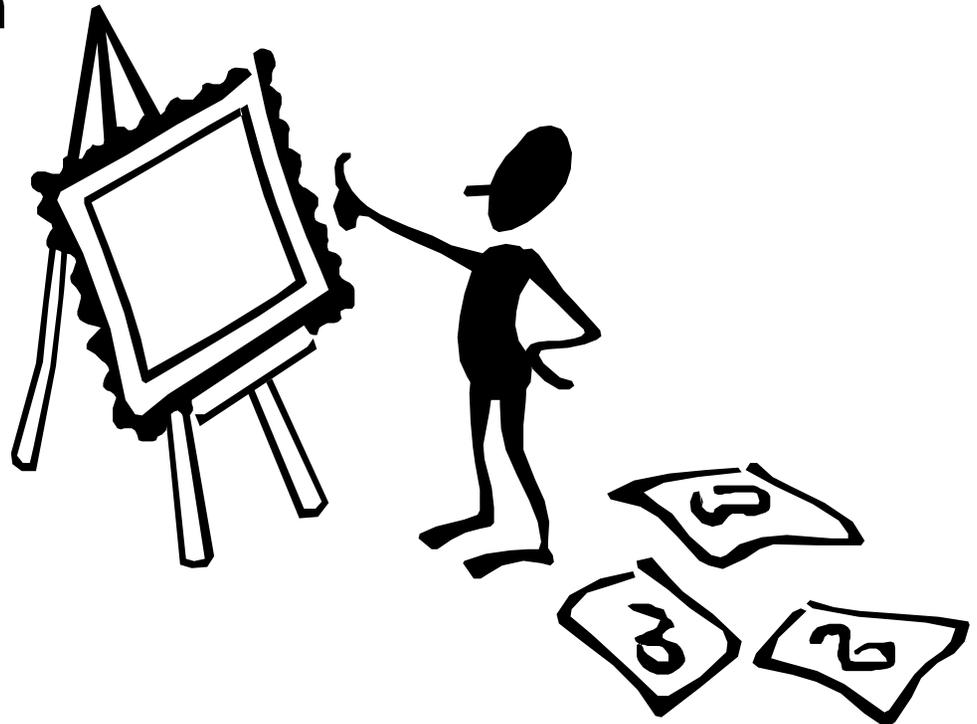


# Mathematische Ausgabe

- MathML momentan wunderschön in Theorie, aber noch nicht ganz praxistauglich:
  - schlechte/unvorhersagbare Browserunterstützung
  - Druckausgabe nach PDF?
  - MathML ist fast unmöglich von Hand zu schreiben
  - Sobald MathML verwendet wird, muss auch der Rest der Seite strikt XML-konform sein, oder viele Renderer brechen ab
- Mathematiker und Physiker sprechen LaTeX
- Aber: Hope springs eternal
- Dann: LaTeX -> MathML weitere Möglichkeit für existierende Inhalte

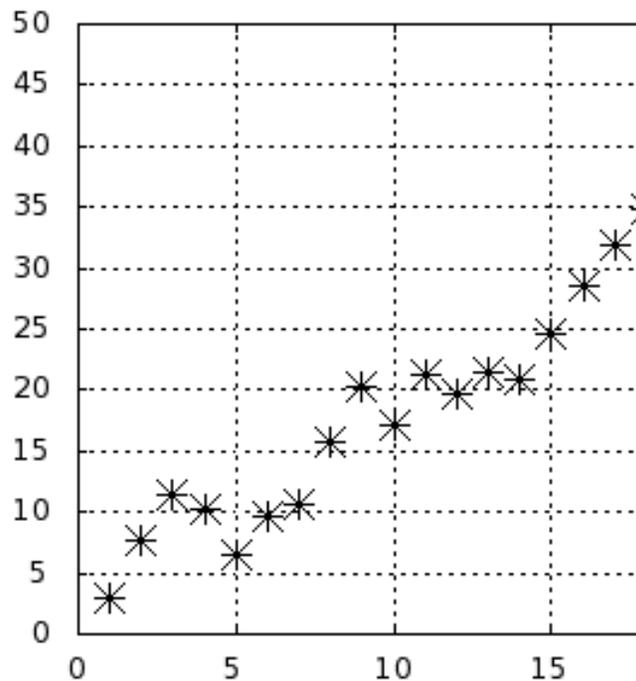
# Mathematische Ausgabe

- Dynamische Graphen
  - Datenpunkte
  - Funktionen
  - Linien-Grafiken
- Verwendet intern GNUplot



# Mathematische Ausgabe

- Datenpunkte



Curve Delete?

Color of curve (x000000)

Name of curve to appear in key

Plot with:

Line width (may not apply to all plot styles)

Line type (may not apply to all plot styles)

Point type (may not apply to all plot styles)

Point size (may not apply to all plot styles)

Point to fill -- for filledcurves

---

Comma or space delimited curve data

Insert:

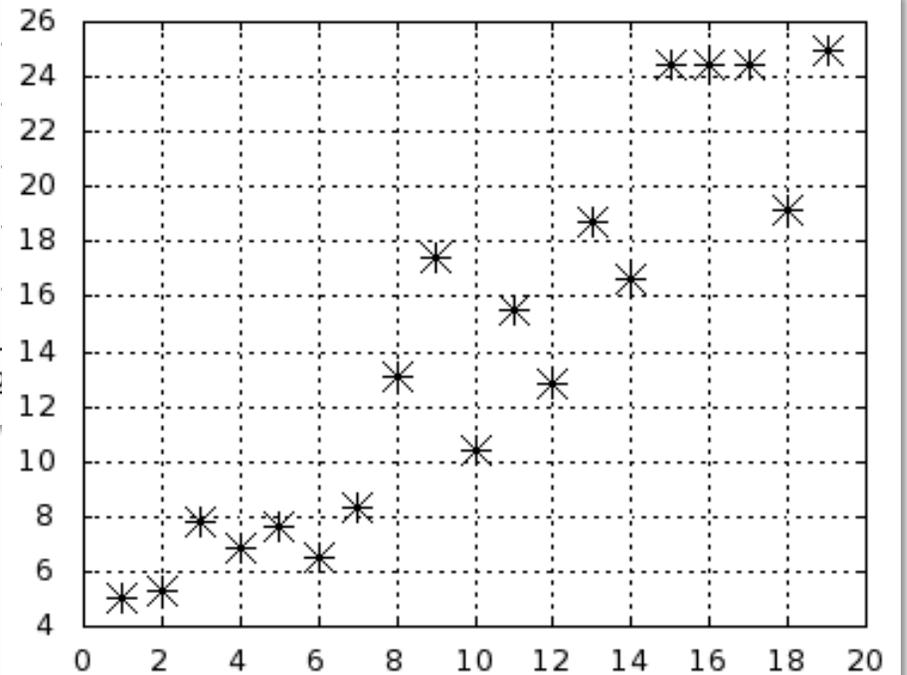
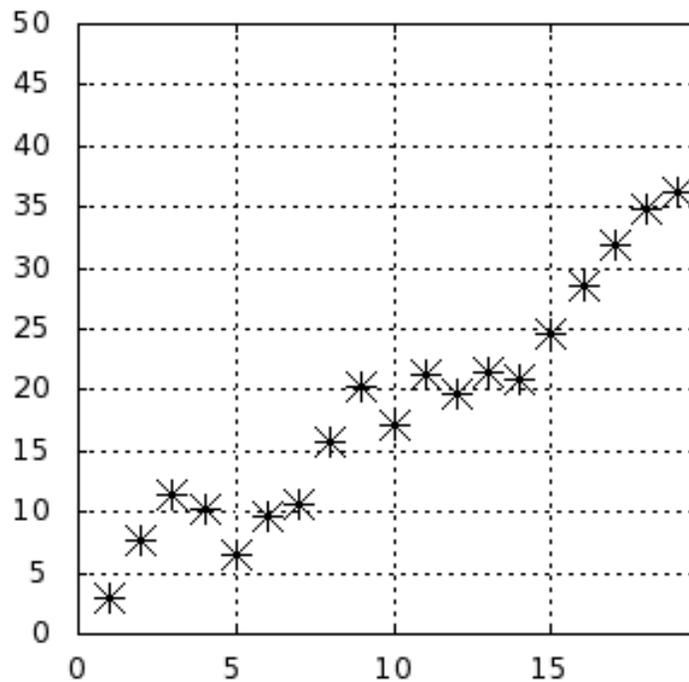
---

Comma or space delimited curve data

Insert:

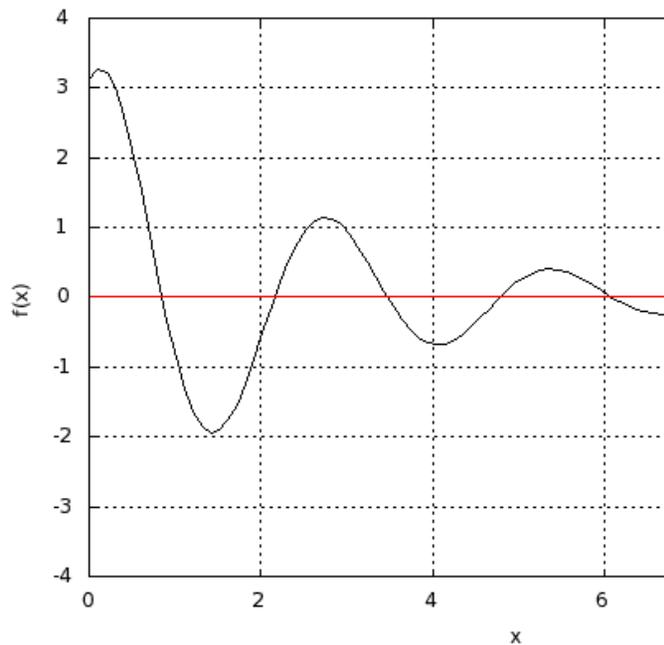
# Mathematische Ausgabe

- Datenpunkte



# Mathematische Ausgabe

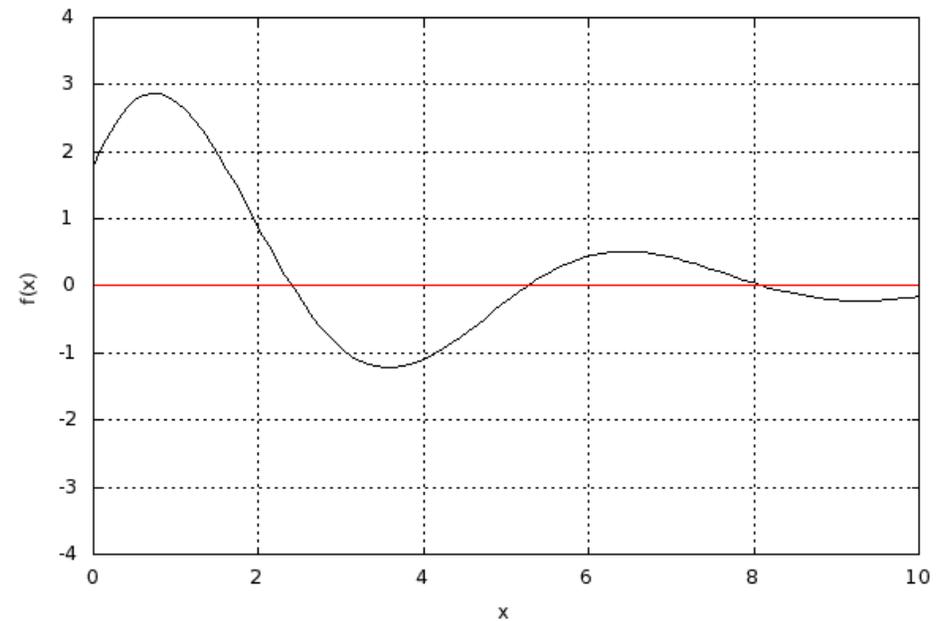
- Funktionen



Match the function indicated in black.

$f(x) =$

Tries 0/99



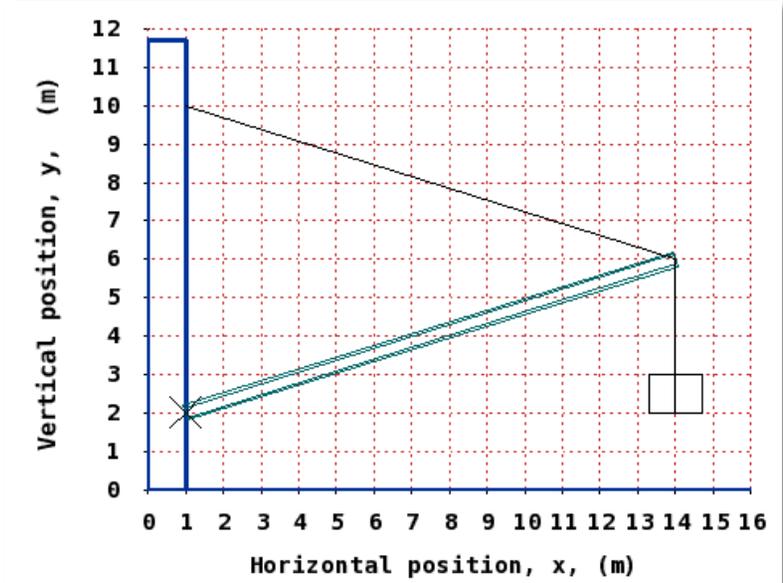
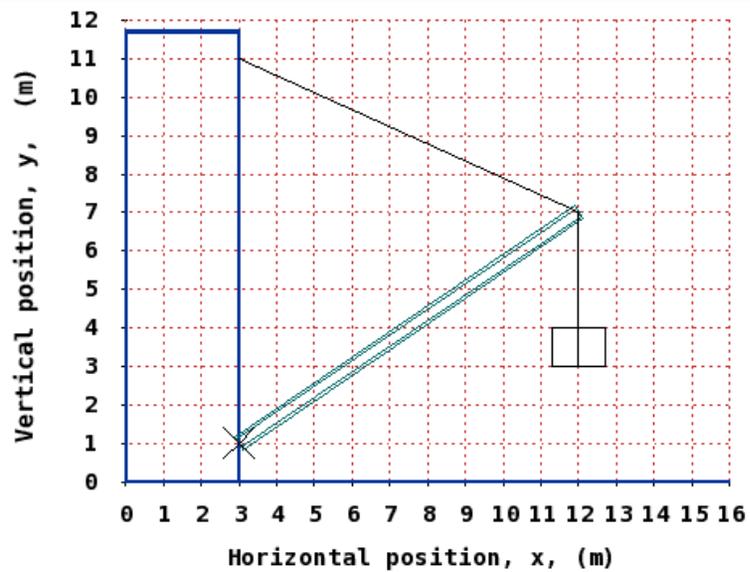
Match the function indicated in black.

$f(x) =$

Tries 0/99

# Mathematische Ausgabe

- Liniengrafiken



# Erzeugung mathematischer Probleme

- Probleme haben zur Verfügung:
  - Perl Scripting Environment
  - MAXIMA Computer Algebra System
  - R Statistics Package
- Problem werden nicht nur einfach irgendwie randomisiert, sondern können mit bestimmten mathematischen Eigenschaften erzeugt werden



# Erzeugung mathematischer Probleme

- Direkter Aufruf von MAXIMA:  
`$result=&cas('maxima',$expression);`
- Einfaches Beispiel: gekürzte Brüche

Script

Delete?



```
# Construct an Egyptian Fraction that can be represented by three terms with denominators between 3 and 12
@denominators=(&random_permutation(&random(1,1000,1), (3..12)))[0..2];
$egyptian='1/' . (join('+1/', sort{$a<=>$b}(@denominators)));
$possible="A possible solution is $egyptian";
```

```
# Let the CAS figure out the value
$solution=&cas('maxima',$egyptian);
```

Write 103/165 as an Egyptian Fraction

Submit Answer

Tries 0

Answer for Part: 0

A possible solution is  $1/3+1/5+1/11$

# Erzeugung mathematischer Probleme

- Direct calls to R:  
\$result=&cas('R',\$expression);  
\$results=&cas\_hashref('R',\$expression);
- Example: generate a distribution with certain properties:

Script

Delete?



```
$seed=&random(1,500,1);  
$n=&random(15,25,1);  
$offset=&random(2,5,0.1);  
$slope=&random(0.6,2.5,0.1);  
# construct a data set using R  
# dump is for debugging, print to screen to see data structure  
($data,$dump)=&cas_hashref('R',"set.seed($seed);x<-1:$n;w<-1+sqrt(x)/2;data.frame(x=x,y=$offset+$slope*x+rnorm(x)*w);");  
@x=&cas_hashref_array($data,'x');  
@y=&cas_hashref_array($data,'y');
```

# Mathematische Eingabe

Script Delete?

```
$vx=&random(3,6,0.1);  
$vy=&random(2,8,0.1);  
$vz=&random(4,10,0.1);  
$t=&random(4,9,1);  
@solution=($vx*$t,$vy*$t,$vz*$t);
```

Insert:

Text Block Delete?

**Rich formatting »**

An object starts at the origin with a constant velocity of

```
<m eval="on">  
\[\vec{v}=\left(\begin{array}{c}$vx\\ $vy\\ $vz\end{array}\right)\right]  
\frac{\mbox{m}}{\mbox{s}}\  
</m>  
Where is it $t seconds later?
```

[Check Spelling](#)

Insert:

Response: Numerical Delete?   Insert:

Answer:  Incorrect Answers:

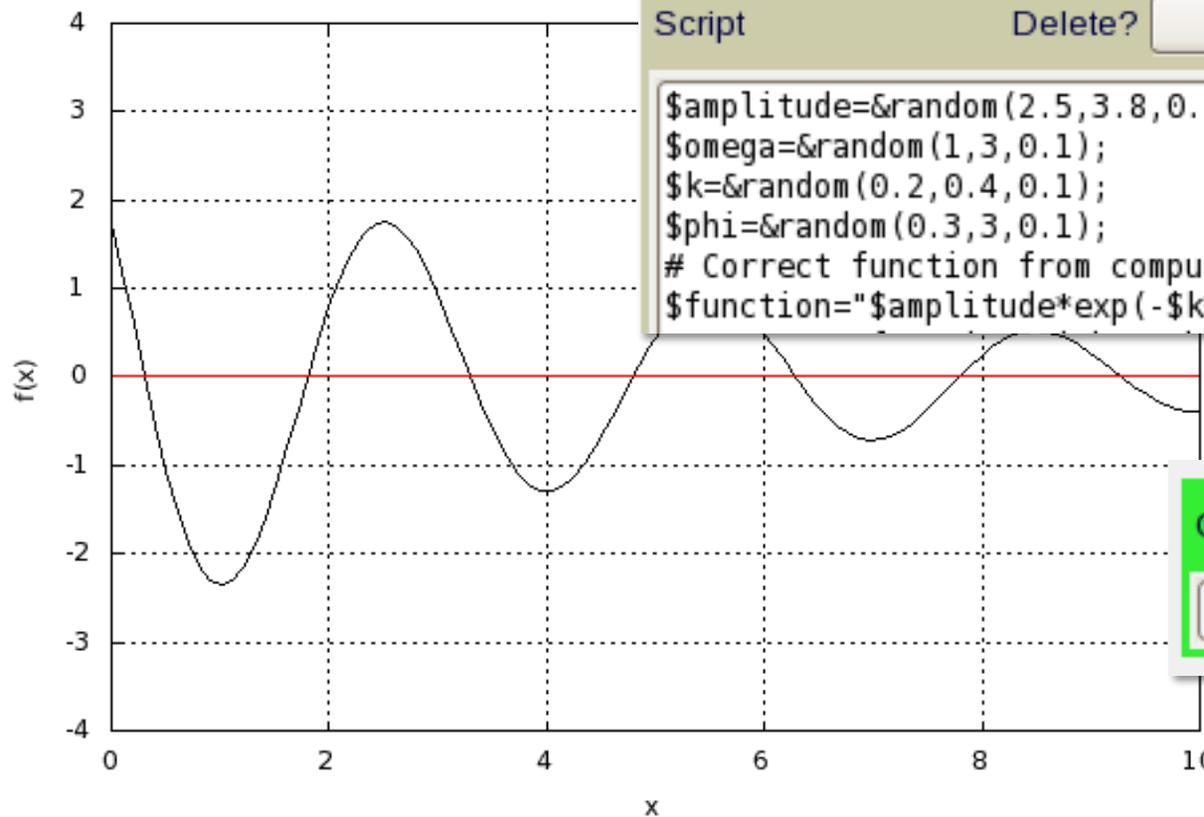
An object starts at the origin with a constant velocity of

$$\vec{v} = \begin{pmatrix} 4.4 \\ 2.5 \\ 7.2 \end{pmatrix} \begin{matrix} \text{m} \\ \text{s} \end{matrix}$$

Where is it 4 seconds later?

- Einfachste Variante: numerisch

# Mathematische Eingabe



Script

Delete?

```
$amplitude=&random(2.5,3.8,0.1);  
$omega=&random(1,3,0.1);  
$k=&random(0.2,0.4,0.1);  
$phi=&random(0.3,3,0.1);  
# Correct function from computer  
$function="$amplitude*exp(-$k*x)*sin($omega*x+$phi)";
```

Gnuplot compatible curve function

\$function

Match the function indicated in black.

f(x)=

Response: Formula

Delete?

Insert:

Answer: \$function

Sample Points: x@0;1;2;3;4;5;6;7;8;9

- Näherung:  
Funktionen

# Mathematische Eingabe

Script Delete?

```
$k=&random(3,6,1);  
$formula="a*x^$k";  
$m=$k-1;  
$derivative="$k*a*x^$m";
```

What is the derivative of  $a \cdot x^4$  with respect to  $x$ ?  

Tries 0

Answer for Part: 0

What is the derivative of  $\langle \text{algebra} \rangle \$formula \langle / \text{algebra} \rangle$  with respect to  $x$ ?

[Check Spelling](#)

rt:

Response: Formula Delete?  Insert:

Answer:  Sample Points:

- Symbolisch: Exakte Antwort

# Mathematische Eingabe

- Eigenschaften prüfen: R

Provide a list of 3 numbers (separated by commas) that has a mean value of 6.2.

Response: Math

Delete?

Insert:

String to display for answer:

Algebra System:

[?](#) Libraries:



Answer algorithm

Delete?

```
x<-c (RESPONSE[1],RESPONSE[2],RESPONSE[3]);  
abs (mean (x) -LONCAPALIST[1])<0.001
```

# Mathematische Eingabe

- Eigenschaften prüfen: MAXIMA

Give an example of a function

1. which is orthogonal to

$$-2 \cdot \cos(5 \cdot x) + 2 \cdot \sin(4 \cdot x)$$

with respect to the scalar product

$$\langle g | h \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} dx g(x) \cdot h(x)$$

2. whose norm is 1.

Submit Answer

Try

Answer algorithm

```
overlap:integrate((RESPONSE[1])*(LONCAPALIST[1]),x,-%pi,%pi)/%pi;  
norm:integrate((RESPONSE[1])*(RESPONSE[1]),x,-%pi,%pi)/%pi;  
is(overlap=0 and norm=1);
```

# Mathematical Input

- Eigenschaften prüfen:  
Perl und MAXIMA

Write 9/20 as an Egyptian Fraction

```
# Subroutine that checks if the provided term is indeed an Egyptian Fraction
sub analyze {
    my ($expression)=@_;
    $expression=~s/\s//gs;
    $expression=~s/\+?1\//\,/gs;
    if ($expression=~/^(\,[0-9]+)+$/ ) {
# Format is indeed 1/n+1/m+...
        $last=-1;
        foreach $number (sort { $a<=>$b } split(/\,/,$expression)) {
# Is a number used twice?
            if ($last==$number) { return(0,1); }
            $last=$number;
        }
        return(0,0);
    }
    return(1,0);
}
```

Answer algorithm

Delete?

```
# Analyze the format
($formaterror,$doubleerror)=&analyze($submission);
if ($formaterror || $doubleerror) { return 'WRONG_FORMAT'; }
# It is an Egyptian Fraction, is the value correct?
if (&cas('maxima',$submission.'-('$egyptian.')) eq '0') {
    return 'EXACT_ANS';
}
return 'INCORRECT';
```

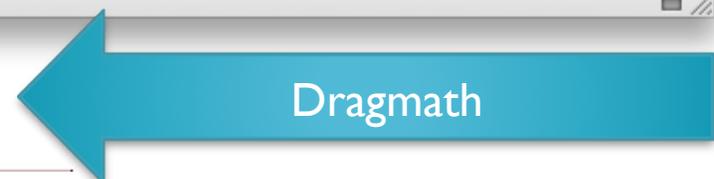
# Mathematische Eingabe

The image shows a screenshot of a web browser window displaying a graphing applet and a math input interface. The browser window is titled "Mozilla Firefox" and shows the URL "https://s10.lite.msu.edu/adm/dragmath/applet/MaximaPopup.html". The applet interface includes a graph of a function  $f(x)$  on a coordinate plane. The x-axis ranges from 0 to 4, and the y-axis ranges from -4 to 4. The function is a smooth curve that starts at (0, 2), reaches a peak at approximately (0.8, 3.1), crosses the x-axis at approximately (1.5, 0), reaches a trough at approximately (2.2, -1.9), crosses the x-axis again at approximately (3.2, 0), and reaches a second peak at approximately (3.8, 1.1). The graphing interface includes buttons for "New Randomization", "Change Random Seed To", "Edit", and "EditXML". The math input interface includes a menu with "File", "Edit", "Options", and "Help", a toolbar with mathematical symbols, and a text input field containing the expression  $e^{-(0.3) \cdot x}$ . A "Save & Close" button is located at the bottom of the input interface. The status bar at the bottom of the browser window indicates "Applet DragMath started".

Match the function indicated in black.

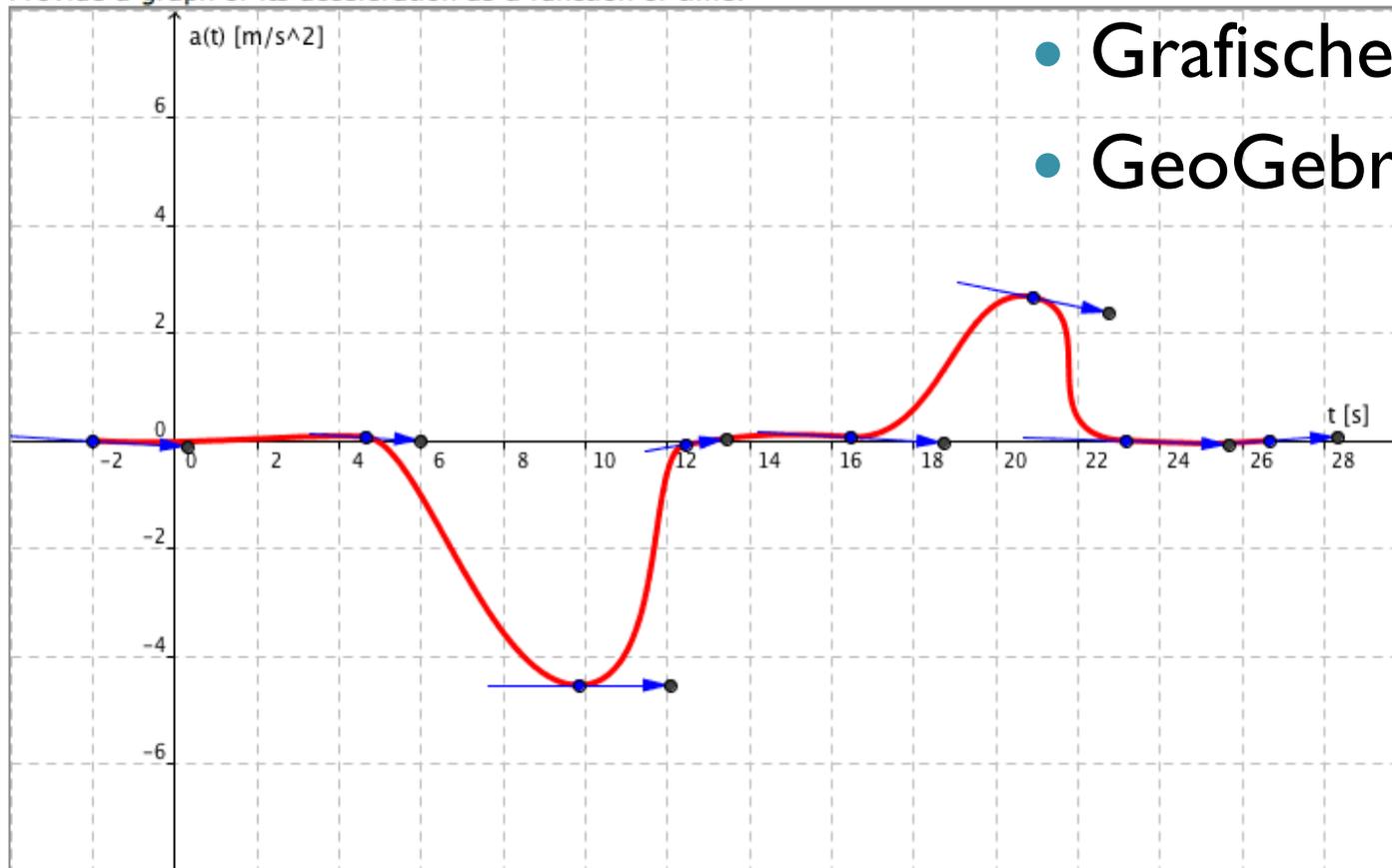
$f(x) =$

Tries 0/99



# Mathematische Eingabe

At  $t=0$  s, a car cruises at a constant positive velocity. Suddenly, a light switches to red. At  $t=10$  s, the driver is maximum on the brake. The car then stops in front of the red light for over 2 seconds. Eventually, it drives off, and then again cruises at a constant velocity. The car cannot accelerate with more than  $3 \text{ m/s}^2$ . Provide a graph of its acceleration as a function of time.



- Grafische Eingabe
- GeoGebra

**You are correct.** Computer's answer now shown above. [Previous Tries](#)

[Script Vars](#)  
[Rules Log](#)

# Mathematical Input

- Grafische Eingabe:  
Regeln

Function  
First Derivative  
Second Derivative  
Integral

Function Plot Evaluation Rule Delete?  Evaluatio

Index/Name:  Function:

Initial x-value:  Initial x-value label:

Final x-value (optional):  Final x-value label (optional):

Minimum length for range (optional):  Maximum length for range (optional):

Relationship:

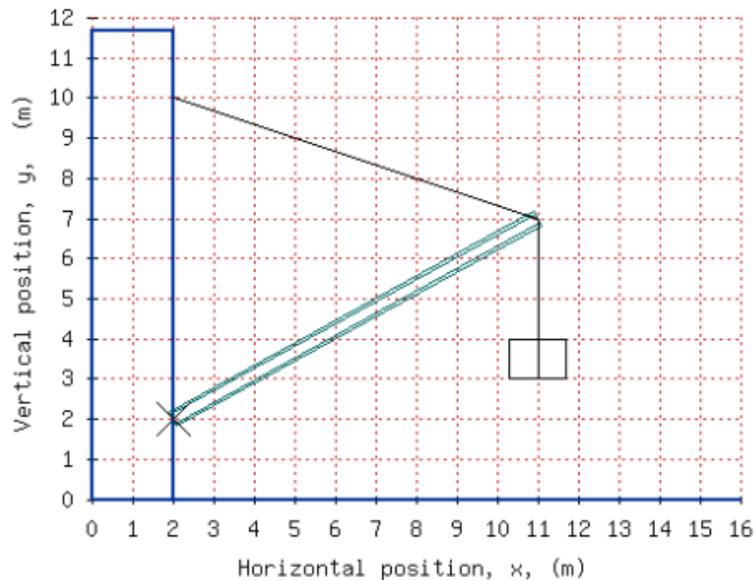
Percent error:

Symbolic, computed, or  
hard-coded ranges

# Mathematische Eingabe

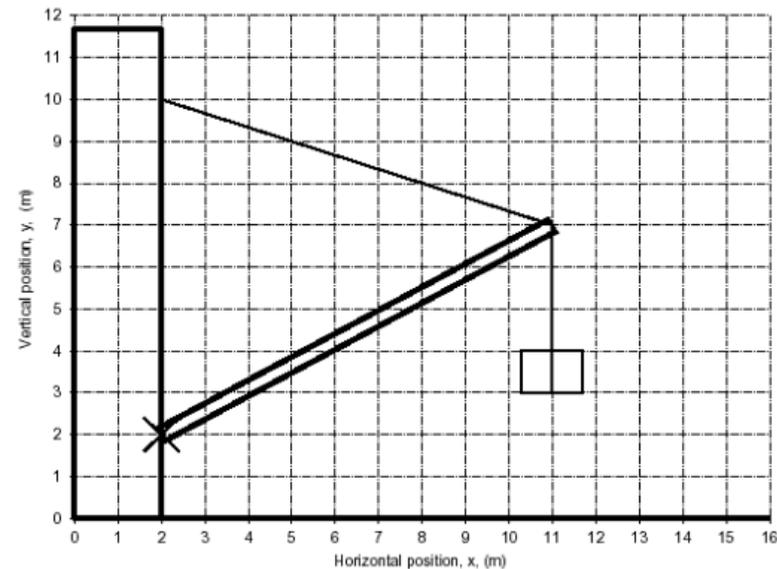
- Rendering Target: Bubblesheet Exams
- Jeder Studierende hat ein anderes Examen

A crate with a mass of 177.5 kg is suspended from the end of a uniform boom with mass of 88.5 kg. The upper end of the boom is supported by a cable attached to the wall and the lower end by a pivot (marked X) on the same wall. Calculate the tension in the cable.




Submit Answer

1 pt A crate with a mass of 177.5 kg is suspended from the end of a uniform boom with mass of 88.5 kg. The upper end of the boom is supported by a cable attached to the wall and the lower end by a pivot (marked X) on the same wall. Calculate the tension in the cable.



(in N)

22. A   $2.58 \times 10^3$     B   $2.92 \times 10^3$     C   $3.29 \times 10^3$   
 D   $3.72 \times 10^3$     E   $4.21 \times 10^3$     F   $4.75 \times 10^3$   
 G   $5.37 \times 10^3$     H   $6.07 \times 10^3$

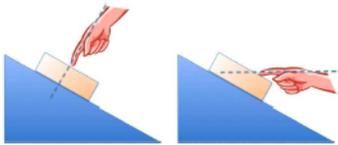
# Mathematische Eingabe

CODE - AACHDA  
LB 271 - Introductory Physics Lecture  
Version A

Name:

LB271 Fall 2009 Final Exam Version A

Gravitational Acceleration on Earth	$g = 9.81 \text{ m/s}^2$
Gravitational Constant	$G = 6.67 \cdot 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2)$
Absolute Zero	$-273.15^\circ\text{C}$
Gas Constant	$R = 8.31 \text{ J}/(\text{K} \cdot \text{mol})$
Boltzmann Constant	$k = 1.38 \cdot 10^{-23} \text{ J/K}$
Avogadro's number	$N_A = 6.02 \cdot 10^{23} \text{ particles/mol}$
Specific heat of water vapor	$c_{\text{vapor}} = 0.48 \text{ kcal}/(\text{kg} \cdot \text{K})$
Specific heat of liquid water	$c_{\text{water}} = 1 \text{ kcal}/(\text{kg} \cdot \text{K})$ $= 4186 \text{ J}/(\text{kg} \cdot \text{K})$
Specific heat of water ice	$c_{\text{ice}} = 0.5 \text{ kcal}/(\text{kg} \cdot \text{K})$
Latent heat of fusion for water	$L_f = 80 \text{ kcal/kg}$
Latent heat of vaporization for water	$L_v = 540 \text{ kcal/kg}$



A block is being held in place on an incline. The magnitude of the force applied by the hand on the block is the same in the left and the right scenarios.

**1 pt** In which scenario does the incline exert a lower normal force on the block?

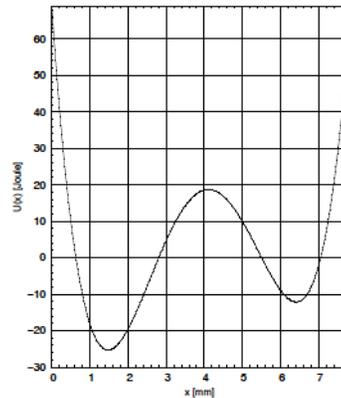
1.  A The left scenario.  
 B The right scenario.  
 C Same in both scenarios.

**1 pt** In which scenario does the incline exert a lower frictional force on the block?

2.  A The left scenario.  
 B The right scenario.  
 C Same in both scenarios.

**1 pt** By how many decibels does the sound intensity from a point source decrease if you increase the distance to it by a factor 6?

3.  A 12.2     B 13.8     C 15.6     D 17.6  
 E 19.9     F 22.5     G 25.4     H 28.7



**1 pt** A particle is located at  $x=2.0$  mm and has a kinetic energy of 29.5 Joule. What is the maximum x-coordinate the particle could reach? (in mm)

4.  A 0.1     B 0.7     C 1.6     D 2.6  
 E 3.2     F 4.7     G 5.3     H 7.6



Deep Space Nine sees Enterprise and a shuttle approach from exactly opposite directions with  $0.8c$  and  $0.5c$ , respectively.

**1 pt** At what fraction of the speed of light ( $\beta$ ) does Enterprise see the shuttle approach?

5.  A 0.00     B 0.50     C 0.83     D 0.91  
 E 0.93     F 1.00     G 1.25     H 1.30

**1 pt** The shuttle has a length of 9 meters when at rest. How long is it in the system of Deep Space 9? (in m)

6.  A 1.8     B 2.6     C 3.7     D 5.4  
 E 7.8     F 11.3     G 16.4     H 23.8

**1 pt** Captain Picard on the Enterprise takes a 49 minute tea break. How long is this break in the system of Deep Space 9? (in min)

7.  A 27     B 33     C 42     D 52  
 E 65     F 82     G 102     H 128

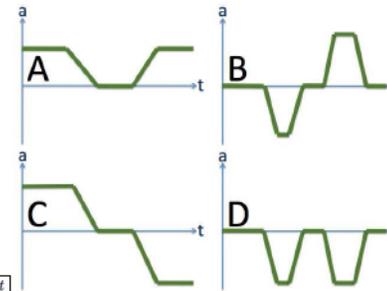
CODE - AACHDA  
LB 271 - Introductory Physics Lecture  
Version A

**1 pt** You have two organ pipes of the same length, one closed at both ends, one half open. Which one has a lower fundamental frequency?

8.  A The closed pipe.  
 B Same.  
 C The half-open pipe.

**1 pt** In a very simple model of the lower atmosphere, air has a constant density of  $1.26 \text{ kg/m}^3$ . How much would the air pressure change over a height difference of 130 m? (in Pa)

9.  A 986     B 1110     C 1260     D 1420  
 E 1610     F 1820     G 2050     H 2320

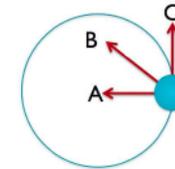


**1 pt** A car drives in the forward (positive) direction. It first has a constant speed, then drives into a parking spot, waits for a few moments, and then drives out again backwards. Which one of the acceleration graphs could describe this scenario?

10.  A Scenario A  
 B Scenario B  
 C Scenario C  
 D Scenario D  
 E None of the above.

**1 pt** A box is sliding uphill as shown. What is the direction of the frictional force on the box?

11.  A Downhill.  
 B Perpendicular to the surface.  
 C Uphill.  
 D None of the above.



An object is rotating on a circular trajectory as shown. The indicated direction A is toward the center of the trajectory, C is tangential to the trajectory. The object is **rotating clockwise and slowing down**.

**1 pt** What could be the direction of the (linear) acceleration?

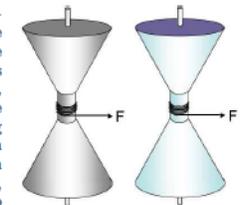
12.  A Direction A.  
 B Direction B.  
 C Direction C.  
 D Into the paper.  
 E Out of the paper.

**1 pt** What could be the direction of the angular acceleration?

13.  A Direction A.  
 B Direction B.  
 C Direction C.  
 D Into the paper.  
 E Out of the paper.

**1 pt**

You have two identical looking spools (same mass, same shape, same size). However, one is hollow, made from iron, the other is solid, made from aluminum. A string is wound around each spool. If you pull on both strings with equal forces, which spool is going to have the larger angular acceleration?



14.  A Same  
 B The solid spool  
 C The hollow spool

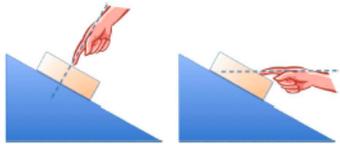
# Mathematische Eingabe

CODE - AAFIHH  
LB 271 - Introductory Physics Lecture  
Version A

Name:

## LB271 Fall 2009 Final Exam Version A

Gravitational Acceleration on Earth	$g = 9.81 \text{ m/s}^2$
Gravitational Constant	$G = 6.67 \cdot 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2)$
Absolute Zero	$-273.15^\circ \text{C}$
Gas Constant	$R = 8.31 \text{ J}/(\text{K} \cdot \text{mol})$
Boltzmann Constant	$k = 1.38 \cdot 10^{-23} \text{ J/K}$
Avogadro's number	$N_A = 6.02 \cdot 10^{23} \text{ particles/mol}$
Specific heat of water vapor	$c_{\text{vapor}} = 0.48 \text{ kcal}/(\text{kg} \cdot \text{K})$
Specific heat of liquid water	$c_{\text{water}} = 1 \text{ kcal}/(\text{kg} \cdot \text{K})$ $= 4186 \text{ J}/(\text{kg} \cdot \text{K})$
Specific heat of water ice	$c_{\text{ice}} = 0.5 \text{ kcal}/(\text{kg} \cdot \text{K})$
Latent heat of fusion for water	$L_f = 80 \text{ kcal/kg}$
Latent heat of vaporization for water	$L_v = 540 \text{ kcal/kg}$



A block is being held in place on an incline. The magnitude of the force applied by the hand on the block is the same in the left and the right scenarios.

**1 pt** In which scenario does the incline exert a higher frictional force on the block?

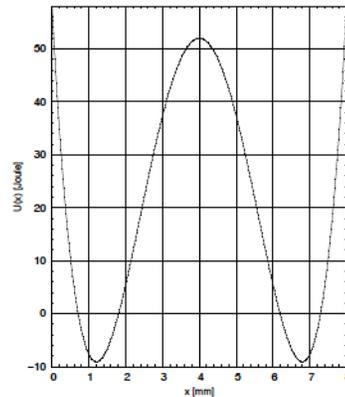
- A The left scenario.
- B The right scenario.
- C Same in both scenarios.

**1 pt** In which scenario does the incline exert a higher normal force on the block?

- A The left scenario.
- B The right scenario.
- C Same in both scenarios.

**1 pt** By how many decibels does the sound intensity from a point source decrease if you increase the distance to it by a factor 6?

- A 7.10
- B 8.31
- C 9.72
- D 11.4
- E 13.3
- F 15.6
- G 18.2
- H 21.3



**1 pt**

A particle is located at  $x = 5.5 \text{ mm}$  and has a kinetic energy of 9.8 Joule. What is the minimum x-coordinate the particle could reach? (in mm)

- A 1.6
- B 2.6
- C 2.7
- D 2.9
- E 3.0
- F 3.8
- G 5.2
- H 6.9



Deep Space Nine sees Enterprise and a shuttle approach from exactly opposite directions with  $0.8c$  and  $0.4c$ , respectively.

**1 pt** At what fraction of the speed of light ( $\beta$ ) does Enterprise see the shuttle approach?

- A 0.00
- B 0.47
- C 0.50
- D 0.59
- E 0.78
- F 0.91
- G 1.00
- H 1.20

**1 pt** The shuttle has a length of 12 meters when at rest. How long is it in the system of Deep Space 9? (in m)

- A 3.6
- B 4.5
- C 5.6
- D 7.0
- E 8.8
- F 11.0
- G 13.7
- H 17.2

**1 pt** Captain Picard on the Enterprise takes a 35 minute tea break. How long is this break in the system of Deep Space 9? (in min)

- A 19
- B 28
- C 40
- D 58
- E 85
- F 123
- G 178
- H 258

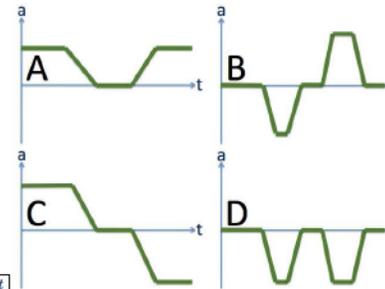
CODE - AAFIHH  
LB 271 - Introductory Physics Lecture  
Version A

**1 pt** You have two organ pipes of the same length, one closed at both ends, one half open. Which one has a lower fundamental frequency?

- A Same.
- B The closed pipe.
- C The half-open pipe.

**1 pt** In a very simple model of the lower atmosphere, air has a constant density of  $1.22 \text{ kg/m}^3$ . How much would the air pressure change over a height difference of 110 m? (in Pa)

- A 1320
- B 1490
- C 1680
- D 1900
- E 2150
- F 2430
- G 2740
- H 3100



**1 pt**

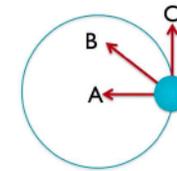
A car drives in the forward (positive) direction. It first has a constant speed, then drives into a parking spot, waits for a few moments, and then drives out again backwards. Which one of the acceleration graphs could describe this scenario?

- A Scenario A
- B Scenario B
- C Scenario C
- D Scenario D
- E None of the above.

**1 pt**

A box is sliding uphill as shown. What is the direction of the frictional force on the box?

- A Perpendicular to the surface.
- B Downhill.
- C Uphill.
- D None of the above.



An object is rotating on a circular trajectory as shown. The indicated direction A is toward the center of the trajectory, C is tangential to the trajectory. The object is **rotating clockwise and slowing down**.

**1 pt** What could be the direction of the (linear) acceleration?

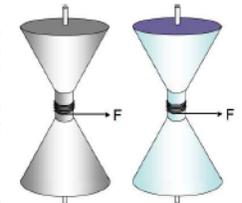
- A Direction A.
- B Direction B.
- C Direction C.
- D Into the paper.
- E Out of the paper.

**1 pt** What could be the direction of the angular acceleration?

- A Direction A.
- B Direction B.
- C Direction C.
- D Into the paper.
- E Out of the paper.

**1 pt**

You have two identical looking spools (same mass, same shape, same size). However, one is hollow, made from iron, the other is solid, made from aluminum. A string is wound around each spool. If you pull on both strings with equal forces, which spool is going to have the larger angular acceleration?



- A The solid spool
- B The hollow spool
- C Same



# Mathematische Eingabe



Resistance

Numerische Eingabe  
von Clickern

Current through a resistor



What is the current in milliampere?

# Mathematische Eingabe

LON-CAPA kann Clickerdaten nach der Vorlesung auswerten

The screenshot shows a web browser window titled "LON-CAPA Change Preferences". The address bar displays the URL "http://phy1.lbs.msu.edu/adm/p". The browser's address bar includes navigation buttons (back, forward, refresh, home) and a search engine (Google). Below the browser window, the LON-CAPA interface is visible, featuring a yellow header with navigation links: "Main Menu", "Launch Remote Control", "Roles", "Help", and "Exit". The main content area is titled "Change Preferences" and includes the user name "Gerd The Kortemeyer" and the role "No Role, Cumulative Privileges". A navigation path is shown: "Menu->Set User Preferences->Register Clicker". The main instruction is "Enter response device ('clicker') numbers", followed by a text input field containing the hexadecimal value "005BC59E". A "Register" button is located below the input field.

LON-CAPA Change Preferences

http://phy1.lbs.msu.edu/adm/p

Getting Started Latest Headlines

LON-CAPA Course Statistics an... LON-CAPA Change Preferences

Main Menu Launch Remote Control Roles Help Exit

**Change Preferences** Gerd The Kortemeyer  
No Role, Cumulative Privileges

Menu->Set User Preferences->Register Clicker Change Preferences

Enter response device ("clicker") numbers

005BC59E

Register

# Mathematische Eingabe

[Main Menu](#)[Return to Last Location](#)[Navigate Contents](#)

**Grading** (msu\_8p96131ebae7b47b8msul1 ss08lbs272)

**Current Resource: Mon, Mar 10th**

**Part: 0** score **Type: numerical**

**Specify a file containing the clicker information for this resource.**

 MonMar10thA.csv

Type:

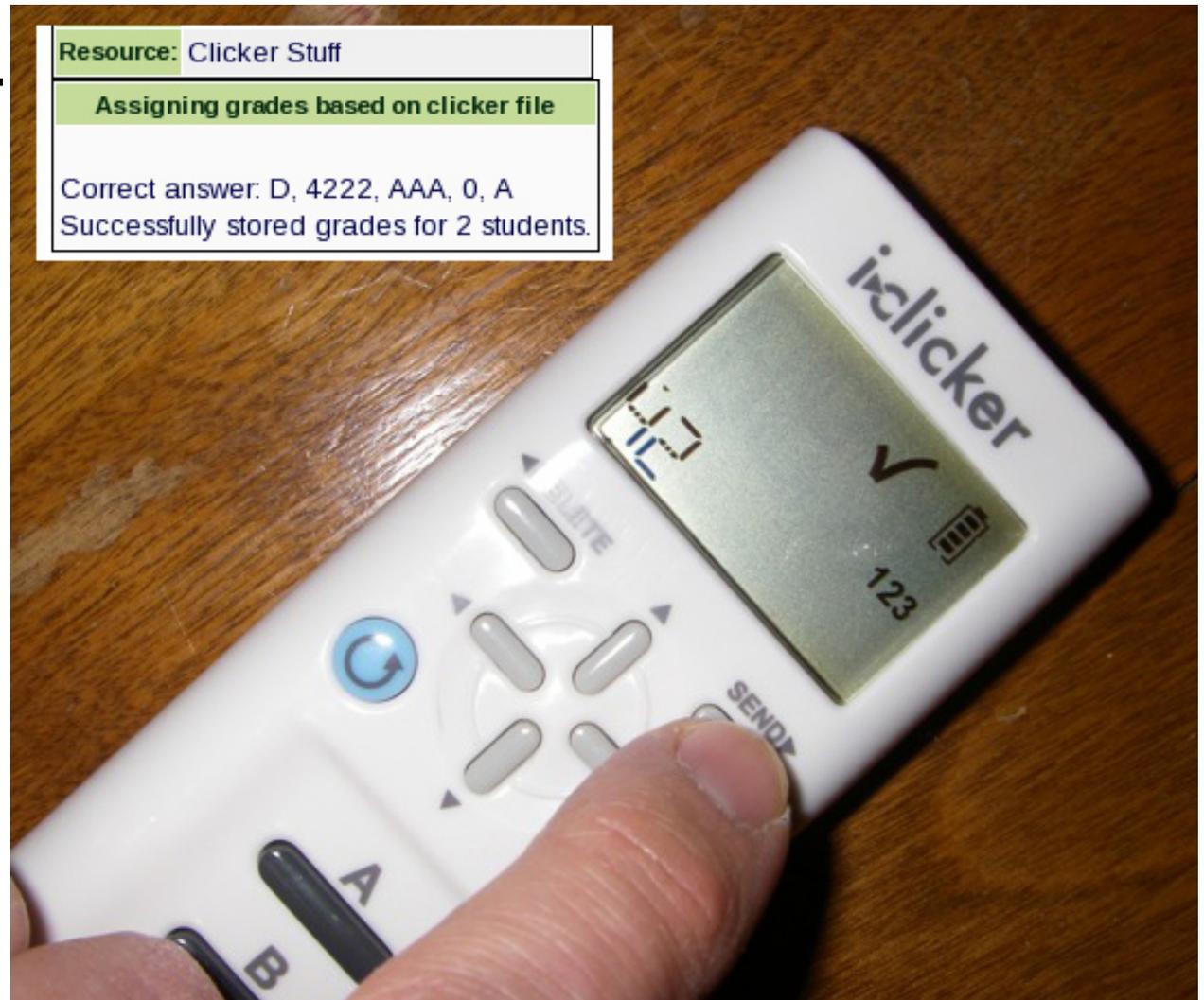
- Award points just for participation
- Correctness determined from response by course personnel
- Correctness determined from response with clicker ID(s)

Percentage points for correct solution:

Percentage points for incorrect solution:

# Mathematische Eingabe

- i>clicker2 integriert in LON-CAPA





# Ausblicke: Beweise

- Derzeit kann man
  - bedingt die Folgerichtigkeit von Beweisen per Computer prüfen,
  - jedoch nicht die Vollständigkeit und
  - nicht die „Eleganz“ oder „Schönheit“
- Einige simple Ansätze in LON-CAPA:  
Beweise nachvollziehen



# Beweise nachvollziehen

- Veselin Jungic, Simon Fraser University (Kanada, nicht USA!)
- Beweise werden in der Vorlesung vorgestellt
- Studierende müssen diese oder ähnliche Beweise als Hausübung in LON-CAPA nachvollziehen
- Ergänzung zu, nicht Ersatz von schriftlichen Hausübungen

# Beweise nachvollziehen

Fill in the blanks in the proof of the following theorem.

**THEOREM:** Let  $f$  be continuous on  $[a, b]$  and suppose that  $f(x) \geq 0$  for all  $x \in [a, b]$ . Prove that if  $\int_a^b f = 0$ , then  $f(x) = 0$  for all  $x \in [a, b]$ .

**Note.** We use as a fact that if  $f$  is a bounded function on  $[a, b]$  such that  $f(x) \geq 0$  for all  $x \in [a, b]$  then  $\int_a^b f \geq 0$ .

**Proof:** Let  $\int_a^b f = 0$  and suppose that there is  $c \in [a, b]$  so that  $f(c) = a > 0$ . Since  $f$  is  there is  $\delta > 0$  such that

$$\forall x \in [a, b], |x - c| < \delta \Rightarrow |f(x) - a| < a/2.$$

Since  $|f(x) - a| < a/2$  is equivalent to

$$-a/2 < f(x) - a < a/2,$$

it follows that for all  $x \in [a, b]$ ,  $|x - c| < \delta$   that  $f(x) > a/2$ .

Let  $[d, e] \subseteq [a, b] \cap (c - \delta, c + \delta)$  be so that  $c \in [d, e]$ . From  $a \leq d < e \leq b$  it follows that

$$\int_a^b f = \int_a^d f + \int_d^e f + \int_e^b f.$$

Since  $f(x) \geq 0$  for all  $x \in [a, d] \cup [e, b]$  we have that  $\int_a^d f + \int_e^b f \geq$  .

Tries 0/3

Since  $f(x) > a/2$  for all  $x \in [d, e]$ , by definition of the  integral we have that

$$\int_d^e f \geq (e - d) \cdot \min\{f(x) : x \in [d, e]\} > (e - d) \cdot a/2 > 0.$$

Thus,

$$\int_a^b f = \int_a^d f + \int_d^e f + \int_e^b f > 0, \text{ which } \text{contradicts} \text{ our assumption that } \int_a^b f = 0.$$

Therefore, if  $f$  is continuous and  $f(x) \geq 0$  for all  $x \in [a, b]$  then  $\int_a^b f = 0$  implies that  $f(x) = 0$  for all  $x \in [a, b]$ .

Tries 0/3

# Beweise nachvollziehen

Gerd Kortemeyer (Course Coordinator)

Test Course VIPP (More ...)

Messages Roles Help Logout

Main Menu | Course Contents | Course Editor | Groups | Switch course role to...

Course Contents » Limits of Functions: Quotient



Functions

Modify user grades for this assessment resource

Modify parameter settings for this resource

Fill in the blanks in the proof of the following theorem.

**THEOREM:** Let  $f:D \rightarrow \mathbf{R}$ ,  $g:D \rightarrow \mathbf{R}$ , and let  $c$  be an accumulation point of  $D$ . If  $\lim_{x \rightarrow c} f(x)=L$  and  $\lim_{x \rightarrow c} g(x)=M$  with  $g(x) \neq 0$  for all  $x \in D$  and  $M \neq 0$ , then

$$\lim_{x \rightarrow c} (f/g)(x)=L/M.$$

**Notes:**

(a) We use as a fact that if  $\lim_{x \rightarrow c} F(x)=l$  and  $\lim_{x \rightarrow c} G(x)=m$  then  $\lim_{x \rightarrow c} (FG)(x)=lm$ .

(b) We use as a fact that from  $\lim_{x \rightarrow c} g(x)=M \neq 0$  it follows that

$$\exists \delta_0 > 0 \quad \ni \quad (0 < |x-c| < \delta_0 \Rightarrow |g(x)| > |M|/2).$$

(**Prove** this statement.)

**Proof:** First we consider the case  $f(x)=1$  for all  $x \in D$ . This means that  $L=$   and we have

$$|1/g(x) - 1/$$
   $| =$    $|g(x)-M|/(|M||g(x)|) <$    $[(2|g(x)-M|)/(M^2)]$

if  $0 <$    $< \delta_0$  where  $\delta_0$  is determined in **Note(b)**.

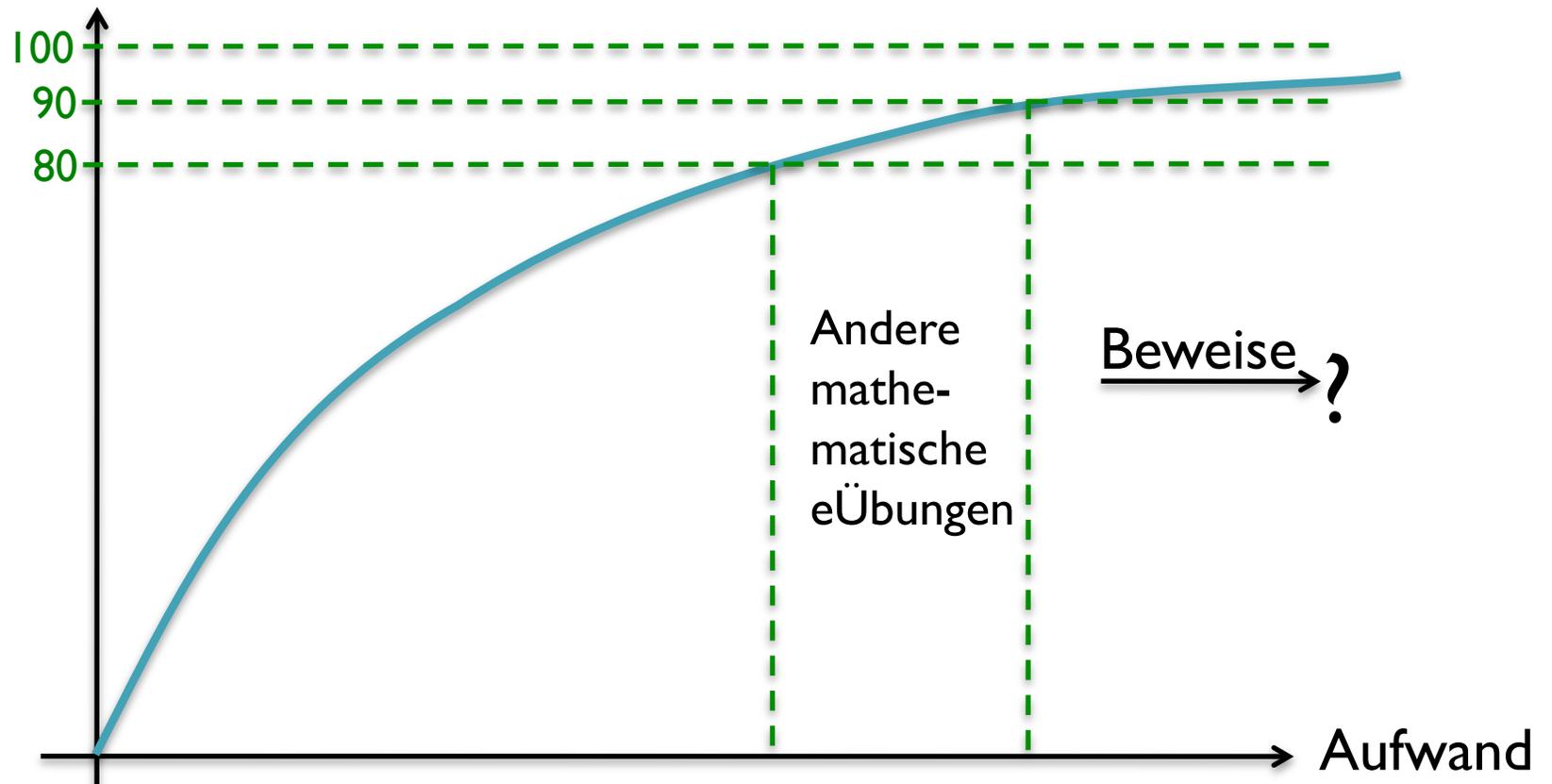
Let  $\epsilon$    $0$ . Since  $\lim_{x \rightarrow c} g(x)=$   , there exists  $\delta_1 > 0$  so that  $|g(x)-M|$    $[(\epsilon M^2)/2]$  if  $0 <$    $< \delta_1$ .

Submit Answer Tries 0/3

# Beweise und eLearning

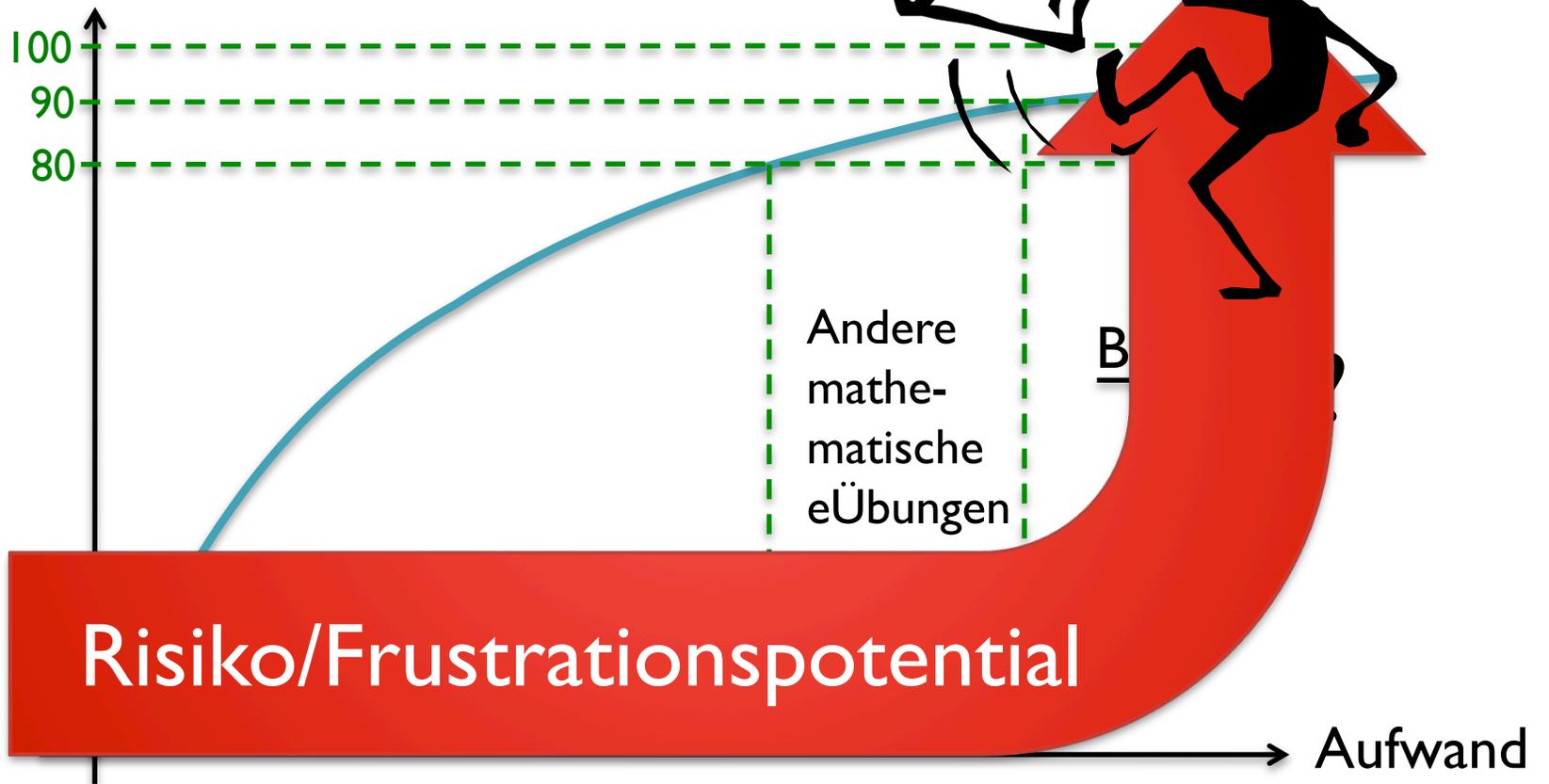
Prozent  
gewünschte  
Funktionalität

## “Diminishing Return”



# Beweise und eLearning

Prozent  
gewünschte  
Funktionalität



# 80% < pragmatisch < 100%

- Mathematischer Typensatz
  - obwohl am CERN erfunden, bietet das Web kaum brauchbare Unterstützung von mathematischem Typensatz
- Aufgaben: mathematische Äquivalenz von Ausdrücken
  - nicht einfach nur Vergleich von Zeichenketten
- Aufgaben: mehr als eine Lösung
  - häufig müssen Ausdrücke nur bestimmte Bedingungen erfüllen
- Aufgaben: schwer zu randomisieren
  - Variation der Aufgaben benötigt symbolische Manipulationen
- Aufgaben: Antwort soll eine bestimmte Form haben
  - Beispiel: schreibe Antwort als Produkt zweier ungerader Polynome
  - Geht über rein mathematische Äquivalenz hinaus
- Beweise:
  - haben eine zentrale Rolle in der Mathematik, sind aber hochgradig offen und nur sehr bedingt algorithmisch auswertbar
  - trotz strikter Logik kreativ und „menschlich“



# Fazit IM MOMENT für LON-CAPA

- Computer macht, was der Computer am besten kann
  - Routineaufgaben
- Menschen machen das, was sie am besten machen – mit mehr Zeit dafür
  - Beweise





**Danke!**

**Gerd Kortemeyer**  
Lyman Briggs College  
and

**Department of Physics and Astronomy**

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<http://www.lon-capa.org/>